Factors and Area Summary

KEY TERMS

- numeric expression
- equation
- Distributive Property
- parallelogram
- altitude
- variable
- trapezoid
- kite
- common factor
- relatively prime
- greatest common factor (GCF)
- multiple
- Commutative Property of Multiplication
- least common multiple (LCM)

LESSON 1 Taking Apart Numbers and Shapes

A **numeric expression** is a mathematical phrase that contains numbers and operations. An **equation** is a mathematical sentence that uses an equals sign to show that two quantities are the same as one another.

There are many ways to rewrite equivalent expressions using properties. The **Distributive Property** states that for any numbers a, b, and c, $a(b + c) = ab + ac$.

For example, you can use the Distributive Property to rewrite the expression $4(2 + 15)$.

$$4\,(2 + 15) = 4 \cdot 2 + 4 \cdot 15$$

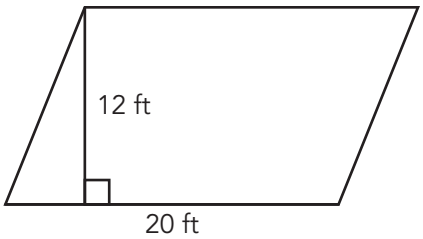
LESSON 2

All About That Base . . . and Height

A **parallelogram** is a four-sided figure with two pairs of parallel sides, with each pair equal in length.

The **altitude** is the height of a geometric figure. In a parallelogram, it is the perpendicular distance from the base to the opposite side. The area of a parallelogram is equal to $b \cdot h$, where the variable b represents the base and h represents the height. A **variable** is a letter that is used to represent a number.

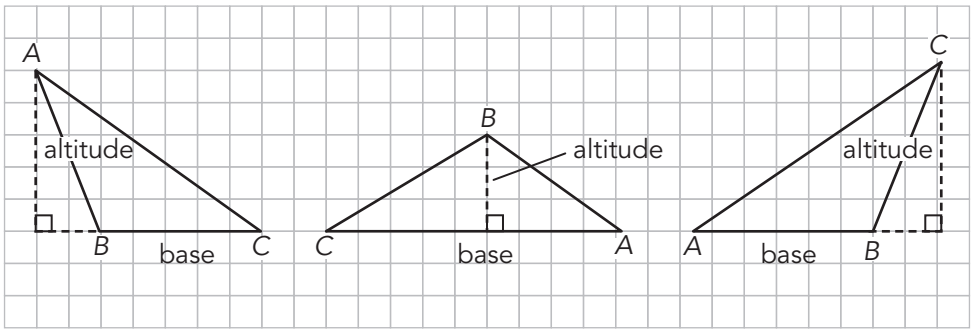
For example, in this parallelogram, the base, b, is 20 feet and the altitude, or height, h, is 12 feet.



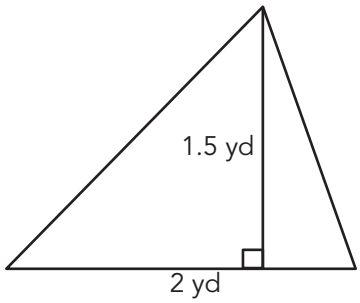
$$\begin{aligned} \text{Area of a parallelogram} &= bh \\ &= (20)(12) \\ &= 240 \text{ square feet} \end{aligned}$$

The area of a triangle is equal to $\frac{1}{2}bh$. The base of a triangle can be any of its sides.

The height, or altitude, of a triangle is the length of a line segment drawn from a vertex of the triangle to the opposite side so that it forms a right angle with the opposite side.



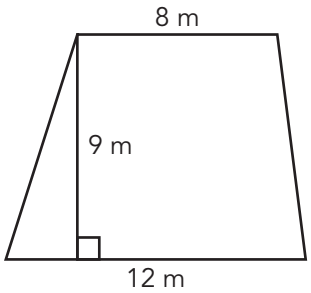
For example, in this triangle, the base, b, is equal to 2 yards and the altitude, or height, h, is equal to 1.5 yards.



$$\begin{aligned} \text{Area of a triangle} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2}(2)(1.5) \\ &= 1.5 \text{ square yards} \end{aligned}$$

A **trapezoid** is a quadrilateral with two bases, often labeled b_1 and b_2. The bases are parallel to each other. The height is the perpendicular distance between the bases. The area of a trapezoid is equal to $\frac{1}{2}(b_1 + b_2)h$.

For example, in this trapezoid, one of the bases is 8 meters and the other base is 12 meters. The altitude, or height, h, of the trapezoid is 9 meters.



$$\begin{aligned}\text{Area of trapezoid} &= \tfrac{1}{2}(b_1 + b_2)h \\ &= \tfrac{1}{2}(8 + 12)(9) \\ &= \tfrac{1}{2}(20)(9) \\ &= 90 \text{ square meters}\end{aligned}$$

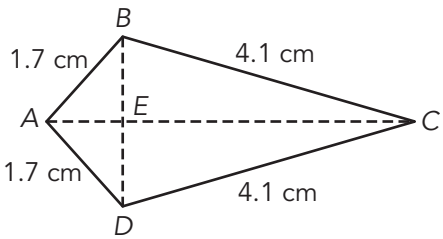
LESSON 3

Slicing and Dicing

A **kite** is a quadrilateral with two pairs of consecutive congruent sides where opposite sides are not congruent. The area of a kite, like that of other quadrilaterals, can be determined by decomposing its shape into smaller familiar shapes.

Area is additive. The area of a composite figure can be determined by decomposing it into familiar shapes and then adding together the areas of those shapes.

For example, in Kite *ABCD*, $BE = ED = 1.25$ cm, and $AC = 5$ cm.



$$\begin{aligned}\text{Area of Kite } ABCD &= \text{Area of Triangle } ABC + \\ &\quad\ \text{Area of Triangle } ADC \\ &= \tfrac{1}{2}(5)(1.25) + \tfrac{1}{2}(5)(1.25) \\ &= 3.125 + 3.125 \\ &= 6.25\end{aligned}$$

The area of Kite *ABCD* is 6.25 square centimeters.

LESSON 4

Searching for Common Ground

When two or more numbers are factored, any factors that the numbers share are **common factors**. Two numbers that do not have any common factors other than 1 are called **relatively prime**. The **greatest common factor (GCF)** is the largest factor two or more numbers have in common.

One way to determine common factors is using prime factorization.

For example, you can use prime factorization to determine common factors of 56 and 42. Start by writing each number as a product of its prime factors.

$56 = 2 \cdot 2 \cdot 2 \cdot 7$
$42 = 2 \cdot 3 \cdot 7$

Organize the prime factors into a table, where only shared factors are listed in the same column.

Number	Prime Factors				
56	2	2	2		7
42	2			3	7

The common factors of the two numbers are the numbers that are in both rows and the product of the numbers that are in both rows.

The common factors of 56 and 42 are 2, 7, and 14.

The greatest common factor of 56 and 42 is 14.

A **multiple** is the product of a given whole number and another whole number. The **Commutative Property of Multiplication** states that for any numbers a and b, the product $a \cdot b$ is equal to the product $b \cdot a$. The first eight multiples of 6 and 8 are given.

6: 6, 12, 18, 24, 30, 36, 42, 48
8: 8, 16, 24, 32, 40, 48, 56, 64

The **least common multiple (LCM)** is the smallest multiple (other than zero) that two or more numbers have in common. The LCM of 6 and 8 is 24.

LESSON 5

Composing and Decomposing Numbers

The greatest common factor is the product of the common prime factors.

For example, the GCF of 24, 40, and 72 is $2 \times 2 \times 2 = 8$.

Number	Prime Factors					
24	2	2	2	3		
40	2	2	2		5	
72	2	2	2	3		3

When using GCF and LCM to solve problems, remember that common factors help you think about how to divide, or share things equally, and common multiples help you think about how things with different cycles can occur at the same time.

For example, a local bus arrives at the stop near Aaron's house every 15 minutes. An express bus arrives at the same stop every 9 minutes. Aaron sees both a local and an express bus arrive at the stop at 10 A.M. What is the next time that he would expect to see both buses arrive at the stop?

The problem is asking about when the two different cycles of the buses will occur again at the same time, so you can use the least common multiple of 15 and 9 to answer the question.

The multiples of 15 are 15, 30, 45, 60, 75, . . .
The multiples of 9 are 9, 18, 27, 36, 45, 54, . . .

The least common multiple of 15 and 9 is 45, therefore the two buses should arrive at the stop at the same time every 45 minutes. The next time Aaron would expect to see both buses at the stop is 10:45 A.M.