

# Positive Rational Numbers Summary

## KEY TERMS

- positive rational number
- benchmark fraction
- complex fraction
- reciprocal
- multiplicative inverse
- Multiplicative Inverse Property

LESSON  
**1**

## Thinking Rationally

A **positive rational number** is a number that can be written in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are both whole numbers greater than 0.

Any decimal greater than 0 that has a limited number of digits after the decimal point (like 0.5) or whose digits repeat in a pattern (like 0.3333 . . .) is a positive rational number.

For example, is 0.75 a rational number?

To write a decimal like 0.75 in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are both whole numbers and  $b$  is not equal to 0:

- Read the decimal using place value.

0.75 —→ seventy-five hundredths

- Write the decimal as a fraction.

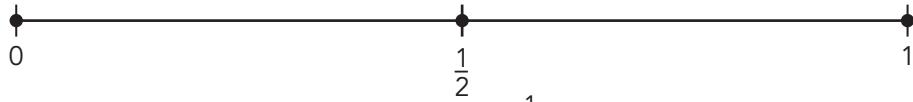
0.75 —→  $\frac{75}{100}$

The fraction  $\frac{75}{100}$  is written in the form  $\frac{a}{b}$ , where  $a$  is equal to 75 and  $b$  is equal to 100. The numbers 75 and 100 are both whole numbers greater than 0.

So, 0.75 is a rational number.

**Benchmark fractions** are common fractions you can use to estimate the value of fractions.

Three common benchmark fractions are  $\frac{0}{1}$ ,  $\frac{1}{2}$ , and  $\frac{1}{1}$ .



A fraction is close to 0 when the numerator is very small compared to the denominator.

A fraction is close to  $\frac{1}{2}$  when the numerator is about half the size of the denominator.

A fraction is close to 1 when the numerator is very close in size to the denominator.

An inequality is a statement that one number is less than or greater than another number.

For example, write an inequality comparing the fractions  $\frac{7}{8}$ ,  $\frac{1}{4}$ , and  $\frac{2}{5}$ .

#### Using Benchmark Fractions

$\frac{7}{8}$  is close to the benchmark  $\frac{1}{1}$ , or 1.

#### Using Equivalent Fractions

$$\frac{7}{8} = \frac{35}{40}$$

$\frac{1}{4}$  is close to the benchmark  $\frac{0}{1}$ , or 0.

$$\frac{1}{4} = \frac{10}{40}$$

$\frac{2}{5}$  is close to the benchmark  $\frac{1}{2}$ .

$$\frac{2}{5} = \frac{16}{40}$$

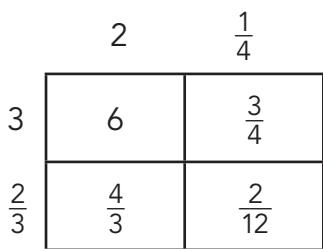
So,  $\frac{1}{4} < \frac{2}{5} < \frac{7}{8}$ .

## LESSON 2

### Did You Get the Part?

You can use area models to multiply mixed numbers or you can write the mixed numbers as improper fractions before multiplying. For example, calculate the product of  $3\frac{2}{3} \times 2\frac{1}{4}$ .

#### Area Model



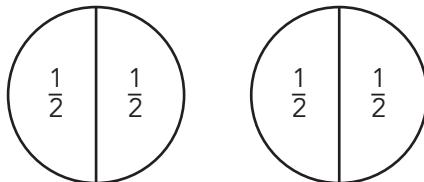
$$\begin{array}{r} 6 \\ \frac{3}{4} = \frac{9}{12} \\ \frac{4}{3} = \frac{16}{12} \\ + \frac{2}{12} = \frac{2}{12} \\ \hline = 6 + \frac{27}{12} = 6 + 2\frac{3}{12} \\ = 6 + 2\frac{1}{4} = 8\frac{1}{4} \end{array}$$

#### Improper Fractions

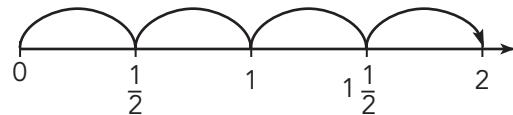
$$\begin{aligned} 3\frac{2}{3} \times 2\frac{1}{4} &= \frac{11}{3} \times \frac{9}{4} \\ &= \frac{99}{12} \\ &= \frac{33}{4} \\ &= 8\frac{1}{4} \end{aligned}$$

Division often means to ask how many groups of a certain size are contained in a number. When you divide with fractions, you are asking the same question. Examine the models shown.

**Physical Model**



**Number Line Model**

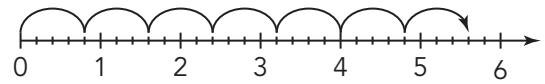


There are four  $\frac{1}{2}$  parts in 2, so  $2 \div \frac{1}{2} = 4$ .

The expression  $2 \div \frac{1}{2}$  is asking how many halves are in 2.

In another example, the expression  $6 \div \frac{4}{5}$  can mean, "How many groups of  $\frac{4}{5}$  are in 6?"

There are 7 whole groups of  $\frac{4}{5}$  in 6 and what is left over is half of a group of  $\frac{4}{5}$ . So,  $6 \div \frac{4}{5} = 7\frac{1}{2}$ .



**LESSON  
3**

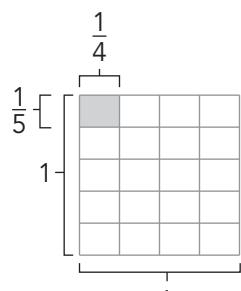
## Yours Is to Reason Why!

You can write multiplication-division fact families for models involving fractions.

For example, the shaded area represents the fraction  $\frac{1}{20}$ . The height of the shaded rectangle is  $\frac{1}{5}$  the height of the model and the width of the shaded rectangle is  $\frac{1}{4}$  the width of the model.

So, the shaded area of the rectangle represents the product  $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$ .

Therefore,  $\frac{1}{20} \div \frac{1}{5} = \frac{1}{4}$  and  $\frac{1}{20} \div \frac{1}{4} = \frac{1}{5}$ .



You can also use fraction strip models to represent fraction division. For example, this model shows  $\frac{3}{4} \div \frac{1}{4}$ . The division expression asks, how many  $\frac{1}{4}$ s are in  $\frac{3}{4}$ ?



$$\frac{3}{4} \div \frac{1}{4} = 3$$

You can “divide across” to determine the quotient of two fractions.

For example, determine the quotient:  $\frac{7}{8} \div \frac{1}{2}$ .

Divide the numerators. Then divide the denominators.  $\frac{7}{8} \div \frac{1}{2} = \frac{7 \div 1}{8 \div 2} = \frac{7}{4}$

You may sometimes write a complex fraction while dividing across. A **complex fraction** is a fraction that has a fraction in either the numerator, the denominator, or both the numerator and denominator. You can use the reciprocal of a number to change a complex fraction to a rational number.

The **reciprocal** of a number is also known as the multiplicative inverse of the number. The **multiplicative inverse** of a number  $\frac{a}{b}$  is the number  $\frac{b}{a}$ , where  $a$  and  $b$  are nonzero numbers.

The **Multiplicative Inverse Property** states that  $\frac{a}{b} \times \frac{b}{a} = 1$ , where  $a$  and  $b$  are nonzero numbers.

Another way to determine the quotient of two fractions is to multiply by the reciprocal of the divisor.

$$\begin{aligned}\frac{3}{4} \div \frac{1}{3} &= \frac{3}{4} \times \frac{3}{1} \\ &= \frac{\cancel{3}}{4} \times \frac{3}{\cancel{1}} \\ &= \frac{9}{4}\end{aligned}$$

$$\begin{aligned}\frac{3}{4} \div \frac{1}{3} &= \frac{3}{4} \times \frac{3}{1} \\ &= \frac{9}{4} = 2\frac{1}{4}\end{aligned}$$

You can use any of these methods to divide mixed numbers as well.

For example, if you have  $5\frac{2}{3}$  pounds of trail mix, how many bags can you make so that each bag contains  $1\frac{5}{6}$  pounds? Write a division sentence and then convert both mixed numbers to improper fractions.

$$\begin{aligned}5\frac{2}{3} \div 1\frac{5}{6} &= \frac{17}{3} \div \frac{11}{6} \\ &= \frac{17}{3} \cdot \frac{6}{11} = \frac{34}{11} \\ &= 3\frac{1}{11}\end{aligned}$$

Multiply the dividend by the reciprocal of the divisor.

Write the product as a mixed number.