

# 3

# Second Verse, Same as the First

## Equivalent Expressions

### WARM UP

Evaluate each expression.

1.  $5 \div \frac{3}{4}$

2.  $0.24 \div 0.6$

3.  $\frac{(14 + 8)}{2}$

4.  $\frac{14}{2} + \frac{8}{2}$

5. What do you notice about the answers to Questions 3 and 4?

### LEARNING GOALS

- Model algebraic expressions with algebra tiles.
- Simplify algebraic expressions using algebra tiles.
- Simplify algebraic expressions using the associative, commutative, and distributive properties.
- Apply properties of operations to create equivalent expressions.
- Rewrite expressions as the product of two factors.

### KEY TERMS

- like terms
- Distributive Property
- equivalent expressions

You have evaluated numeric expressions and written and evaluated algebraic expressions. How do you combine algebraic expressions, like you did with numeric expressions, into as few terms as possible?

## Getting Started

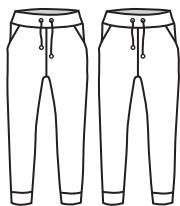
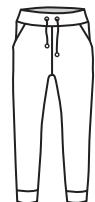
### Packing for a Camping Trip

Jaden and Jerome, twin brothers, are packing for a weekend camping trip. They lay out the following items to go in the suitcase.

**Jaden:**



**Jerome:**



1. How many shirts and pairs of pants is each brother packing?  
Together, how many shirts and pairs of pants are they packing?

Shirts

Pants

**Jaden**

**Jerome**

**Together**

Your teacher has provided you with algebra tiles.

- 2. How can you use algebra tiles to model the number of shirts packed by each brother and the number of shirts they packed together?**



ACTIVITY  
**3.1**

## Algebra Tiles and Combining Like Terms



As you may have seen in the previous activity, when using algebra tiles to model situations and expressions, it is important to have a shared meaning for each differently-sized algebra tile.

Your teacher will hold up each differently-sized algebra tile and tell you the conventional value of each.

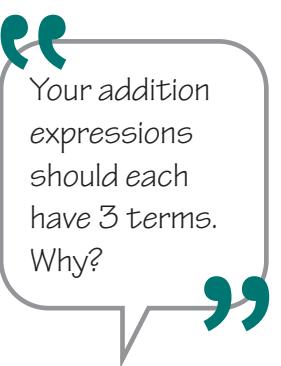
- 1. Sketch each tile and record its value.**

- 2. Represent each numeric or algebraic expression using algebra tiles. Write an addition expression that highlights the different tiles used in the model. Then, sketch the model below the expression.**

a.  $3$

b.  $3x$

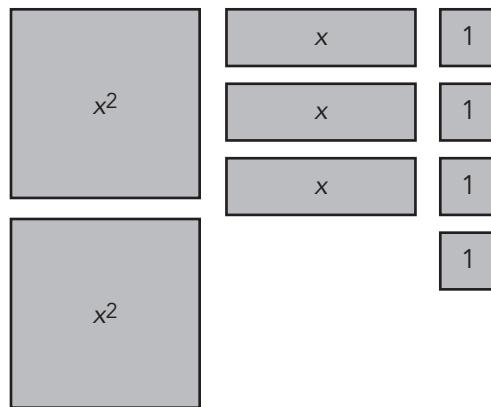
c.  $3x^2$



In an algebraic expression, **like terms** are two or more terms that have the same variable raised to the same power. The coefficients of like terms can be different. Let's start our exploration of combining like terms with a review of the properties of arithmetic and algebra that you will use to combine terms.

The expression you wrote in each part of Question 2 was made up of like terms. All tiles that are the same size and have the same value represent like terms.

- Given the algebra tile model, write an addition expression that highlights the different tiles in the model. Then, if necessary, combine like terms and write the expression using as few terms as possible.



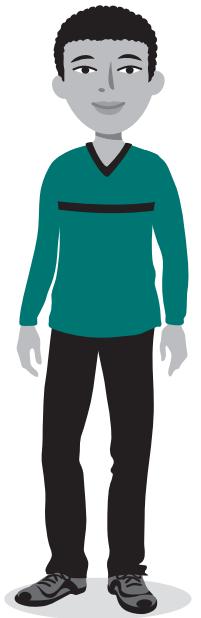
When I combine like terms using models, I just group all the same tiles together.

- Analyze the last expression you wrote in Question 3.

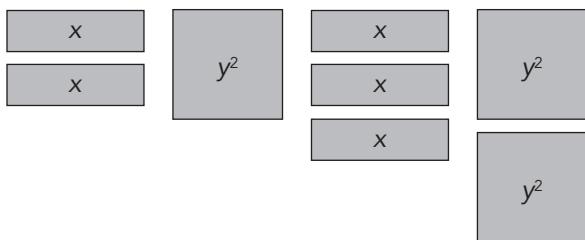
a. How many terms are in your expression with the fewest terms? How does this relate to the algebra tile model?

b. What is the greatest exponent in the expression?

c. What is the coefficient of  $x$  in the expression? How does this relate to your algebra tile model?



## 5. Consider the model.



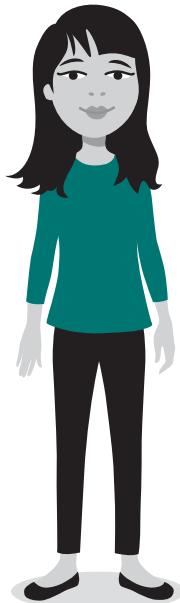
- a. Write an addition expression that highlights the different tiles in the model.
  - b. Rearrange the tiles to combine all of the like tiles. How many terms does your expression have now?
  - c. Write the new algebraic expression represented.
6. Represent the algebraic expression  $3x^2 + x + 2$  using algebra tiles. How many types of tiles are needed?

Algebra tiles are helpful tools for combining like terms in algebraic expressions. However, because they only represent whole number tiles, they cannot be used to model all algebraic expressions.

7. Use what you have learned about combining like terms to rewrite each algebraic expression with as few terms as possible.

- a.  $2x + 3x - 4.5x$
- b.  $3\frac{1}{2}y + 2 + 4y + 1\frac{1}{4}$
- c.  $4.5x + 6y - 3.5x + 7$
- d.  $\frac{3}{4}x + 2 + \frac{3}{8}x$
- e.  $5x + 2y + \frac{1}{3}x^2 - 3x$

So, combining like terms means to add or subtract terms with the same variables. Like  $3x + 5x$ . That's  $8x$ .



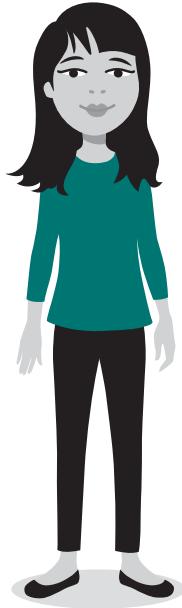
ACTIVITY  
**3.2**

## Algebra Tiles and the Distributive Property



When you are speaking about an algebraic expression that is grouped together with parentheses, use the words "the quantity." For example  $2(x + 3)$  in words would be "two times the quantity  $x$  plus three."

This model is just adding the quantity  $x + 1$  five times!

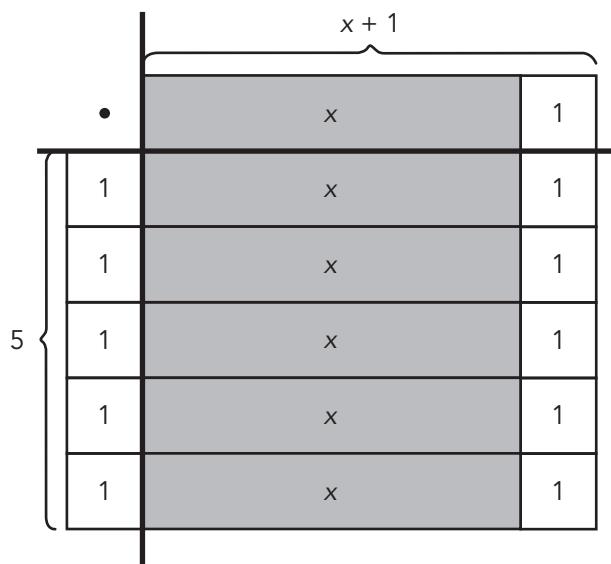


Let's use algebra tiles to explore rewriting algebraic expressions with the Distributive Property.



### WORKED EXAMPLE

Consider the expression  $5(x + 1)$ . This expression has two factors: 5 and the quantity  $(x + 1)$ . You can use the Distributive Property to rewrite this expression. In this case, multiply the 5 by each term of the quantity  $(x + 1)$ . The model using algebra tiles is shown.



$$5(x + 1) = 5x + 5$$

1. Analyze the parts of the mathematical expressions in the worked example. Explain each response.

a. Which expression,  $5(x + 1)$  or  $5x + 5$ , shows a product of two factors?

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b. How many terms are in  $5x + 5$ ?

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c. The number 5 is a coefficient in which expression?

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2. Create a model of each expression using your algebra tiles. Then, sketch the model and rewrite the expression using the Distributive Property.

a.  $4(2x + 1)$

b.  $(3x + 1)2$

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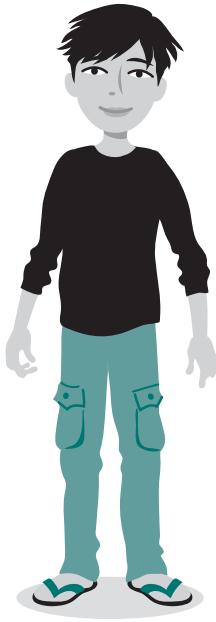
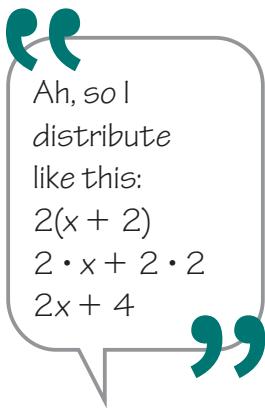
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3. Rewrite each expression using the Distributive Property. Then, combine like terms if possible.

a.  $2(x + 4)$

b.  $\frac{2}{3}(6x + 12)$

c.  $2(x + 5) + 4(x + 7)$

d.  $5x + 2(3x - 7)$

e.  $2(y + 5) + 2(x + 5)$

f.  $\frac{1}{2}(4x + 2) + 8x$

So far in this activity, you have multiplied expressions together using the Distributive Property. Now let's think about how to divide expressions.

How do you think the Distributive Property will play a part in dividing expressions? Let's find out.

4. Consider the expression  $(4x + 8) \div 4$ , which can also be rewritten as  $\frac{4x + 8}{4}$ .

a. First, represent  $4x + 8$  using your algebra tiles. Sketch the model you create.

b. Next, divide your algebra tile model into four equal groups. Then, sketch the model you created with your algebra tiles.

c. Write an expression to represent each group from your sketches in part (b).

d. Verify you created equal groups by multiplying your expression from part (c) by 4. The product you calculate should equal  $4x + 8$ .



Let's consider the division expression from Question 4.

To rewrite the expression, divide the denominator into both terms in the numerator.

### WORKED EXAMPLE

You can rewrite an expression of the form  $\frac{4x + 8}{4}$  using the Distributive Property.

$$\begin{aligned}\frac{4x + 8}{4} &= \frac{4x}{4} + \frac{8}{4} \\ &= 1x + 2 \\ &= x + 2\end{aligned}$$

$$\text{So, } \frac{4x + 8}{4} = x + 2$$

The model you created in Question 4 is an example that shows that the Distributive Property can be used with division as well as with multiplication.

5. Consider the expression  $\frac{2x + 6y + 4}{2}$ .

a. Use algebra tiles to represent the division expression.

b. Rewrite the division expression using the Distributive Property. Then, simplify the expression.

$$\frac{2x + 6y + 4}{2} = \frac{2x}{\boxed{\phantom{0}}} + \frac{6y}{\boxed{\phantom{0}}} + \frac{4}{\boxed{\phantom{0}}}$$

c. Verify that your answer is correct.

$$= \underline{\hspace{2cm}}$$

Zachary thinks he can simplify algebraic expressions that use the Distributive Property with division without using algebra tiles. He wants to rewrite  $\frac{6 + 3(x + 1)}{3}$  in as few terms as possible and proposes two different methods.

**6. Analyze each correct method.**

**Method 1**



$$\begin{aligned}\frac{6 + 3(x + 1)}{3} &= \frac{6}{3} + \frac{3(x + 1)}{3} \\ &= 2 + (x + 1) \\ &= x + 3\end{aligned}$$

**Method 2**

$$\begin{aligned}\frac{6 + 3(x + 1)}{3} &= \frac{6 + 3x + 3}{3} \\ &= \frac{3x + 9}{3} \\ &= \frac{3x}{3} + \frac{9}{3} \\ &= x + 3\end{aligned}$$



**a. Explain the reasoning used in each method.**

**b. Which method do you prefer. Why?**

ACTIVITY  
**3.3**

## Factoring Algebraic Expressions



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Using the Distributive Property to write an expression as a product of two factors is also known as *factoring*.

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You have used the Distributive Property to multiply and divide algebraic expressions by a given value. The Distributive Property can also be used to rewrite an algebraic expression as a product of two factors: a constant and a sum of terms.

You can write any expression as a product of two factors. In many types of math problems, you often need the coefficient of a variable to be 1. Let's explore how to use the Distributive Property — without algebra tiles—to rewrite expressions so that the coefficient of the variable is 1.

**1. Consider the expression  $3x + 6$ .**

**a. Identify the coefficient of the variable term.**

**b. Use the Distributive Property to rewrite the expression as the product of two factors: the coefficient and a sum of terms.**

**c. How can you check your work?**

Using the Distributive Property to rewrite the sum of two terms as the product of two factors is also referred to as factoring expressions. In the expression  $3x + 6$ , you factored out the common factor of 3 from each term and rewrote the expression as  $3(x + 2)$ . In other words, you divided 3 from each term and wrote the expression as the product of 3 and the sum of the remaining factors,  $(x + 2)$ .

You can use the same strategy to rewrite an algebraic expression so that the coefficient of the variable is 1 even if the terms do not have common factors.

### WORKED EXAMPLE

Let's rewrite the expression  $4x - 7$  so the coefficient of the variable is 1.

To rewrite the expression, factor out the coefficient 4 from each term. The equivalent expression is the product of the coefficient and the difference of the remaining factors.

$$\begin{aligned}4x - 7 &= 4\left(\frac{4x}{4} - \frac{7}{4}\right) \\&= 4\left(x - \frac{7}{4}\right)\end{aligned}$$

Remember, you can multiply or divide any expression by 1 and not change its value.

**2. Use the Distributive Property to check that the new expression is equivalent to the original expression in the Worked Example.**

**3. Rewrite each expression as the product of two factors.  
Check your answers.**

a.  $4x + 5$

b.  $8x - 3$

c.  $\frac{1}{2}x - 4$

d.  $1.1x + 1.21$

ACTIVITY  
**3.4**

## Simplifying Algebraic Expressions



Rewrite each expression using the Distributive Property.

1.  $\frac{32 + 4x}{4}$

2.  $15x - 10$

3.  $\frac{3(x + 1) + 12}{3}$

4.  $2\frac{1}{2} + \frac{1}{4}x$

Rewrite each algebraic expression in as few terms as possible.

5.  $3x + 5y - 3x + 2y$

6.  $4x^2 + 4y + 3x + 2y^2$

7.  $7x + 5 - 6x + 2$

8.  $x^2 + 5y + 4x^2 - 3y$

Rewrite each algebraic expression by applying the Distributive Property and then combining like terms.

$$9. \ 4(x + 5y) - 3x$$

$$10. \ 2(2x + 5y) + 3(x + 3y)$$

$$11. \ 3x + 5(2x + 7)$$

$$12. \ \frac{4x + 6y}{2} - 3y$$

$$13. \ 3(x + 2y) + \frac{3x - 9y}{3}$$

$$14. \ 2(x + 3y) + 4(x + 5y) - 3x$$

## TALK the TALK

### Write Right

Mr. Martin asked his class to write expressions equivalent to  $7(3a + 5b)$  and  $8 + 3(2x + 5)$  and got 5 different responses for each. For each response, determine if the original expression was rewritten correctly. For those not rewritten correctly, describe the mistake that was made in rewriting the expression.

1.  $7(3a + 5b)$

- a.  $10a + 12b$
- b.  $7(3a) + 7(5b)$
- c.  $21a + 5b$
- d.  $21a + 35b$
- e.  $7(8ab)$

2.  $8 + 3(2x + 5)$

- a.  $8 + 3 \cdot 2x + 3 \cdot 5$
- b.  $23 + 6x$
- c.  $11(2x + 5)$
- d.  $8 + 6x + 15$
- e.  $13 + 6x$

# Assignment

## Write

Describe 3 different ways that you can use the Distributive Property to rewrite expressions. Provide an example for each.

## Remember

To rewrite an algebraic expression with as few terms as possible, use the properties of arithmetic and the Order of Operations.

An algebraic expression containing terms can be written as the product of two factors by applying the Distributive Property.

## Practice

- Represent each algebraic expression by sketching algebra tiles. Rewrite the expression in a fewer number of terms, if possible.
  - $x^2 + 2y^2 + 5$
  - $y^2 + 3y + 1 + y$
- Rewrite each expression by combining like terms.
  - $4.5x + (6y - 3.5x) + 7$
  - $\left(\frac{2}{3}y + \frac{5}{8}x + \frac{1}{4}\right) + \left(\frac{1}{4}x + \frac{1}{2}\right)$
- Nelson is going on an overnight family reunion camping trip. He is in charge of bringing the wood for the campfire. He will start the fire with 6 logs and then plans to add 3 logs for each hour the fire burns.
  - Represent the number of logs he will use as an algebraic expression.
  - Suppose the family decides to stay for 2 nights next year. Write the expression for the number of logs they would need for 2 nights.
  - Create a model of the situation in part (b) using your algebra tiles, and then sketch the model.
  - Rewrite the expression in part (c) using as few terms as possible.
  - Nelson's cousin believes they will only need one-third of the firewood Nelson brings for one night. Represent this as an expression and then use the Distributive Property to rewrite the expression.
  - There are several family members who will be visiting for the day only. The campground charges \$6 per car, plus \$2 per visitor. One of the families brings a coupon for \$3 off their total fee. Write the expression that represents their total cost for the day. Define the variables.
  - The two oldest uncles at the reunion insist on paying the bill for the daily visitors. They will split the bill equally. Represent the amount of money each uncle will pay as an expression. Then use the Distributive Property to rewrite the expression.
- Rewrite each expression by applying the Distributive Property and combining like terms.
  - $7(2x + y) + 5(x + 4y)$
  - $9x + 6y + \frac{12y + 16x}{4}$
  - $\frac{6(x + 1) + 30}{6}$
- Rewrite each expression as a product of two factors, so that the coefficient of the variable is 1.
  - $6x + 7$
  - $\frac{2}{3}x + 8$

## Stretch

1. Simplify the algebraic expression to include as few terms as possible.

$$3[2x + 4(5y + 1)] + \frac{1}{4}[8y + 12(\frac{2}{3}x + \frac{1}{6})]$$

2. Rewrite each algebraic expression as the product of two factors, such that the coefficient of the term with the highest exponent is 1.

- a.  $2x^2 + 5x + 1$
- b.  $\frac{3}{4}x^3 - 9x^2 + \frac{2}{3}x + 10$
- c.  $2.6y^2 + 3.9y - 12.48$

## Review

1. Sheldon Elementary School has a school store that sells many items including folders, pencils, erasers, and novelty items. The parent association is in charge of buying items for the store.

- a. One popular item at the store is scented pencils that come in packs of 24 from the retailer. Write an algebraic expression that represents the total number of scented pencils they will have available to sell. Let  $p$  represent the number of packs of scented pencils.

- b. Another popular item at the store is animal-themed folders. Each pack of folders contains 6 folders.

The store currently has 4 packs in the store and would like to order more. Write an algebraic expression for the total number of folders they will have after they order more folders. Let  $x$  represent the number of packs of folders they buy.

- c. The latest fad is animal-shaped rubber bracelets. The bracelets come in a pack of 24. Write an algebraic expression that represents the cost of each bracelet. Let  $c$  represent the cost of a pack of 24 bracelets.

2. Determine which rate is faster.

- a. 185 miles in 3 hours or 490 miles in 8 hours
- b. 70 miles per hour or 100 kilometers per hour

3. Calculate the volume of each solid formed by rectangular prisms.

