

# Equations Summary

## KEY TERMS

- equation
- Reflexive Property of Equality
- solution
- Addition Property of Equality
- Subtraction Property of Equality
- Multiplication Property of Equality
- Division Property of Equality
- Symmetric Property of Equality
- Zero Property of Multiplication
- Identity Property of Multiplication
- Identity Property of Addition
- graph of an inequality
- solution set of an inequality
- bar model
- one-step equation
- inverse operations
- literal equation

LESSON  
**1**

## First Among Equals

An **equation** is a statement of equality between two expressions. An equation can contain numbers, variables, or both in the same mathematical sentence. Equations may be always true, never true, or true only for one or more values of the variable. The **Reflexive Property of Equality** says that when both sides of an equation look exactly the same, their values are equal.

A **solution** to an equation is any value for the variable that makes the equation true.

Properties of Equality are logical rules that allow you to maintain balance and rewrite equations.

Always True	Never True	True for certain values of the variable
$6 = 10 - 4$	$10 = 20$	$x = 5$
$x = x$	$x = x + 2$	$x + 2 = 12$

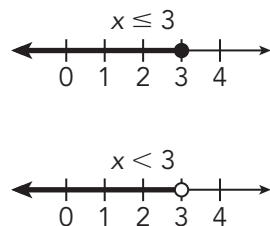
Properties of Equality	For all numbers $a$ , $b$ , and $c$
Addition Property of Equality	If $a = b$ , then $a + c = b + c$ .
Subtraction Property of Equality	If $a = b$ , then $a - c = b - c$ .
Multiplication Property of Equality	If $a = b$ , then $a \cdot c = b \cdot c$ .
Division Property of Equality	If $a = b$ and $c \neq 0$ , then $\frac{a}{c} = \frac{b}{c}$ .
Symmetric Property of Equality	If $a = b$ , then $b = a$ .

Equations that have an infinite number of solutions are equations that are true regardless of the value you assign to the variable. These kinds of equations often describe important properties of numbers. For example:

- The **Zero Property of Multiplication** states that the product of any number and 0 is 0:  $x \cdot 0 = 0$ .
- The **Identity Property of Multiplication** states that the product of any number and 1 is the number:  $x \cdot 1 = x$ .
- The **Identity Property of Addition** states that the sum of any number and 0 is the number:  $x + 0 = x$ .

You can use a number line to represent inequalities. **The graph of an inequality** in one variable is the set of all points on a number line that make the inequality true. The set of all points that make an inequality true is the **solution set of the inequality**.

The solution to any inequality can be represented on a number line by a ray. A ray begins at a starting point and goes on forever in one direction. A closed circle means that the starting point is part of the solution set of the inequality. An open circle means that the starting point is not part of the solution set of the inequality.



For example, the solution set of the inequality  $x \leq 3$  is all numbers equal to or less than 3, and the solution set of the inequality  $x < 3$  is all numbers less than 3.

LESSON  
**2**

## Bar None

A **bar model** uses rectangular bars to represent known and unknown quantities.

For example, the equation  $x + 10 = 15$  states that for some value of  $x$ , the expression  $x + 10$  is equal to 15. This can be represented using a bar model.

$x + 10$
15

The expression  $x + 10$  can be decomposed into a part representing  $x$  and a part representing 10. The number 15 can be decomposed in a similar way:  $15 = 5 + 10$ . The bar model demonstrates that these two equations are equivalent.

$$\begin{aligned}x + 10 &= 15 \\x + 10 &= 5 + 10\end{aligned}$$

$x$	10
$x + 10$	
5	10

By examining the structure of the second equation, you can see that 5 is the value for  $x$  that makes this equation true.

A **one-step equation** is an equation that can be solved using only one operation. To solve a one-step addition equation, isolate the variable using number sense or inverse operations. **Inverse operations** are pairs of operations that reverse the effects of each other.

For example, solve the equation  $h + 6 = 19$ .

$$h + 6 = 13 + 6 \quad \text{Write equivalent expressions that mirror structure.}$$

$$h + 6 - 6 = 13 + 6 - 6 \quad \text{Use inverse operations to reverse the addition of 6 to } h.$$

$$h + 0 = 13 + 0 \quad \text{Combine like terms and apply the Additive Identity Property.}$$

$$h = 13$$

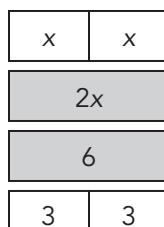
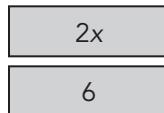
LESSON  
**3**

## Play It In Reverse

You can also use bar models to reason about the solution to multiplication equations.

For example, the equation  $2x = 6$  states that for some value of  $x$ , the expression  $2x$  is equal to 6. You can decompose  $2x$  by rewriting it as the equivalent expression  $1x + 1x$ , or  $x + x$ . To maintain equivalence, decompose 6 in a similar way. The bar model demonstrates that these two equations are equivalent.

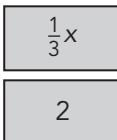
$$\begin{aligned}2x &= 6 \\x + x &= 3 + 3\end{aligned}$$



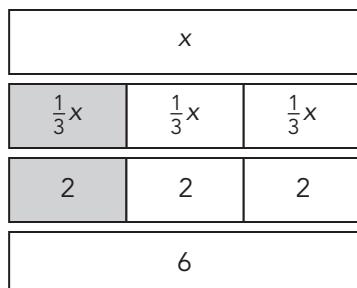
By examining the structure of the second equation, you can see that  $x = 3$ .

Bar models can also be used to solve multiplication equations with fractional coefficients.

For example, represent  $\frac{1}{3}x = 2$  as a bar model.



To solve this equation for  $x$ , compose 3 equally-sized parts to create the whole,  $x$ . To maintain equivalence, compose 3 equally-sized parts for the other expression too. This structure allows you to see the value of  $x$  that makes the equation true:  $x = 6$ .



You can also use the inverse operation of multiplication to solve one-step multiplication equations.

For example, solve the equation  $4r = 32$ .

$$4r = 32$$

$4(1r) = 4(8)$  Write equivalent expressions with similar structure.

$\frac{4(1r)}{4} = \frac{4(8)}{4}$  Use inverse operations to reverse the multiplication of 4 and  $1r$ .

$1r = 1(8)$  Perform division.

$r = 8$  Identity Property of Multiplication

You can use properties of arithmetic and algebra, along with the properties of equality, to solve for one of the variables in an equation in terms of the other variable.

$$12a = 84b$$

Step 1  $12a = (12 \cdot 7)b$

Step 2  $12a = 12(7b)$

Step 3  $a = 7b$

LESSON  
**4**

## Getting Real

**Literal equations** are equations in which the variables represent specific measures. You most often see literal equations when you study formulas. The formula for the area of a triangle,  $A = \frac{1}{2}bh$ , is a literal equation. The variables represent the measures of the base and height of the triangle.

In division problems, the remainder can mean different things in different situations. Sometimes the remainder can be ignored, and sometimes the remainder is the answer to the problem. Sometimes the answer is the quotient without the remainder, and sometimes you need to use the next whole number up from the quotient.

For example, the Red Cross disaster relief fund collected 4233 winter coats to distribute to flood victims. If there are 28 distribution centers, how many coats can be sent to each center?

$$4233 \div 28 = 151\frac{5}{28}$$

You cannot have a fraction of a coat, so each center will receive 151 coats and there will be 5 coats left over.

