Signed Numbers Summary

KEY TERMS

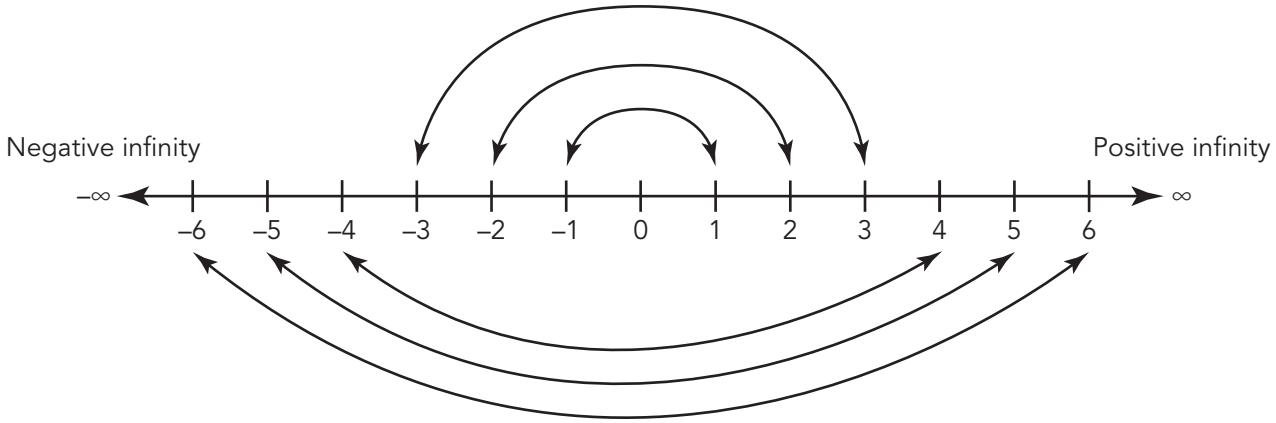
- negative numbers
- infinity
- absolute value
- integers
- ellipsis
- rational numbers
- Density Property

LESSON 1

Human Number Line

A number line can be created by reflecting the positive numbers across zero. The values to the left of zero on the number line are called **negative numbers** and are labeled with a negative sign. You can write a positive number with a positive sign or without any sign. For example, positive 5 can be written as +5 or 5.

The positive values extend to positive infinity, and the negative numbers extend to negative infinity. **Infinity**, represented by the symbol ∞, means a quantity with no end or bound.

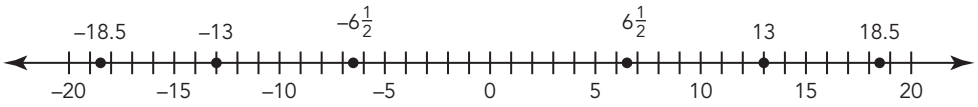


Opposite numbers are reflections of each other across 0 on the number line.

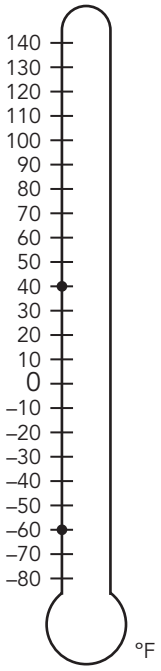
- The opposite of a positive number is a corresponding negative number.
- The opposite of a negative number is a corresponding positive number.

Attaching a negative sign to a number means reflecting that number across zero on the number line. The number 0 is the only number that doesn't have an opposite.

For example, the numbers $6\frac{1}{2}$, -13, -18.5, and their opposites are plotted on the number line.



Number lines can also be vertical, like a thermometer or a measure of elevation.

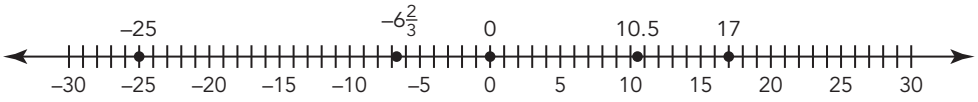


You can use a thermometer to plot temperatures and to compare and order temperatures. In vertical number lines like this one, the greater the value, the higher up on the number line.

For example, compare 40 degrees to -60 degrees. By plotting each temperature on the thermometer, you can see that 40 degrees is above -60 degrees. Therefore $40 > -60$.

You can compare different types of numbers by plotting the numbers on a number line.

For example, the numbers, $-6\frac{2}{3}$, 10.5, -25, 17, and 0 have been plotted on the number line. Use the number line to order the values from least to greatest.



From the number line you can determine that -25 has the least value because it is the farthest to the left and 17 has the greatest value because it is farthest to the right. The numbers ordered from least to greatest are -25, $-6\frac{2}{3}$, 0, 10.5, and 17.

LESSON 2

Magnificent Magnitude

The magnitude, or **absolute value**, of a number is its distance from zero on a number line. The symbol for absolute value is | |. The expression $|n|$ is read as "the absolute value of a number n." Because distance cannot be negative, the absolute value of a number is always positive or 0.

$|9| = 9$, because 9 is 9 units from 0 on a number line.
$|-3.8| = 3.8$, because –3.8 is 3.8 units from 0 on a number line.

Absolute values are used in real-world applications when you are interested in only the number and not in the sign of the number. You also use absolute value statements to describe how numbers compare with other numbers.

Situation	Absolute Value Statement	Numeric Example
The temperature went from 55°F to 5°F.	The temperature fell by 50°F.	−50°F
The bank account balance went from $550 to $795.	The balance increased by $245.	$245
A water level went from 10.3 feet to 6.7 feet.	A water level fell by 3.6 feet.	−3.6 feet
A water level less than $-2\frac{1}{2}$ feet	More than $2\frac{1}{2}$ feet below a full pool	−3 feet
A temperature less than −5° F	Colder than 5°F below 0	−8°F
An account balance less than −$100	A debt greater than $100	−$110

Absolute value equations can be used to calculate the distance between positive and negative numbers to solve real-world problems.

For example, the Top Notch company's bank balances are shown. The table shown represents the first 10 weeks of operation. Overdrafts, which are a negative balance, are represented by amounts within parentheses. What was the gain or loss between Weeks 2 and 3?

Week	1	2	3	4	5	6	7	8	9	10
Balance	$159.25	($201.35)	$231.57	($456.45)	($156)	($12.05)	$281.34	$175	$192.34	$213

At the end of Week 2, the company had a negative balance of $201.35 and at the end of Week 3 it had a positive balance of $231.57. The company had a gain between these two weeks because it went from a lesser balance to a greater balance. The gain is equal to the sum of the absolute values of the two balances.

$$|-\$201.35| + |\$231.57| = \$201.35 + \$231.57 = \$432.92$$

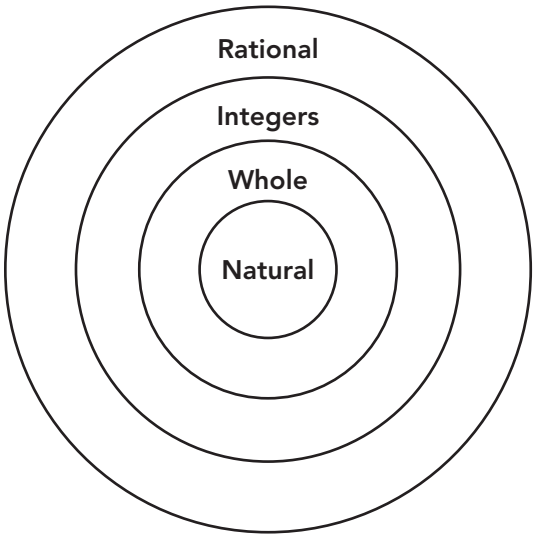
LESSON 3

What's In a Name?

Integers are the set of whole numbers with their opposites. The integers can be represented by the set $\{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$. The three periods before and after the numbers in the set are called an **ellipsis**, and they are used to represent infinity in a number set.

Rational numbers are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and b does not equal 0.

There are many ways you can classify numbers. Many of the classifications are subsets of other classifications. The diagram shows the different sets of numbers you have encountered in your mathematical experiences.



Natural numbers are a subset of whole numbers.

Whole numbers are a subset of integers.

Integers are a subset of rational numbers.

The number 2 is a rational number, an integer, a whole number, and a natural number.

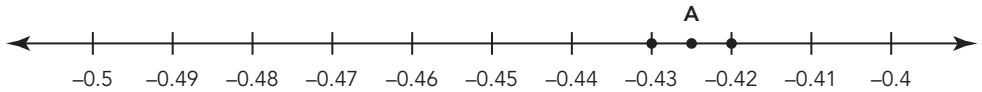
The number 0 is a rational number, an integer, and a whole number.

The number -11 is a rational number and an integer.

The numbers 12.5 and $-\frac{3}{4}$ are both rational numbers.

The **Density Property** states that between any two rational numbers there is another rational number.

For example, consider the rational numbers -0.42 and -0.43 and the number line shown. The number represented by point A is another rational number that falls between -0.42 and -0.43 such that $-0.43 < A < -0.42$. Point A could represent the value -0.425.



The property is not true for natural numbers, whole numbers, or integers. For example, there is no integer between -25 and -26. There is no whole number or natural number between 12 and 13.